

GRADUATE SCHOOL OF BUSINESS ADMINISTRATION

**KOBE UNIVERSITY**

ROKKO KOBE JAPAN

Discussion Paper Series

# Vertical mergers and product differentiation<sup>\*</sup>

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March 10, 2006

## Abstract

This paper presents the development of an equilibrium theory of vertical merger that incorporates strategic behaviors in the Hotelling-type location model for the purpose of considering the relationship between the strategies of downstream firms for product differentiation and vertical integration. I show that vertical integration enhances the degree of product differentiation of the integrated firm. Under some conditions, partial integration appears to be in equilibrium and may increase the profit of the non-integrated downstream firm. Welfare implications of vertical integration are briefly discussed.

**JEL classification numbers:** D43, L13, L22, R32

**Key words:** product differentiation, vertical integration, location model, foreclosure

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<sup>\*</sup> I would like to thank two anonymous referees and the Editor, Professor Yeon-Koo Che, for their helpful comments and suggestions. I would also like to thank the seminar participants at the Institute of Social Science at the University of Tokyo. The author gratefully acknowledges financial support from a Grant-in-Aid for Encouragement of Young Scientists from the Japanese Ministry of Education, Science and Culture. Needless to say, I am responsible for any remaining errors.

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# 1 Introduction

The relation between vertical integration and the competition of downstream firms has been frequently discussed. Ordover *et al.* (1990) and Hart and Tirole (1990) conducted pioneering analyses on vertical integration. In their models, an integrated firm will recognize that it can benefit from the higher costs imposed on its downstream rivals when it refrains from competing aggressively in the input market, and it will thus try to do so to raise the rivals' costs.<sup>1</sup> Vertical foreclosure can therefore arise in equilibrium.<sup>2</sup>

In his comments on Hart and Tirole (1990), Carlton (1990, pp. 278–9) made the following comment on vertical integration: ‘Let me explain how vertical integration can affect the credibility of a commitment. Vertical integration can eliminate opportunism and thereby allow greater specialization of assets to occur. When specialization occurs, product can be more idiosyncratic and can be more differentiated. If products become more differentiated, the force of Bertrand competition can be lessened. Therefore, vertical integration can be a way for firms to commit not to produce identical products, which would be beneficial to them because it would lessen competition.’

As discussed by Carlton, some economists also point to the relation between vertical integration and product differentiation. Porter (1985) pointed out that (forward) vertical integration provides firms with the potential for a differentiation advantage. Perry (1989) commented that (forward) vertical integration can enable a firm to achieve increased differentiation, and, subsequently, safeguard the resulting potential for economic rents. Masten (1984) considered empirically the choice of internal and external organization. Using data from the aerospace industry, he showed that the probability of internalization is higher for complex and highly specialized inputs.

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<sup>1</sup> Salop and Scheffman (1987) and Salinger (1988) provided the basis for the argument for raising a rival's costs.

<sup>2</sup> Reiffen (1992) criticized the model of Ordover *et al.* (1990). In their model, for foreclosure to be in equilibrium, the merged firm must be able to commit not to compete aggressively with the remaining supplier to supply the other downstream firm. Avoiding such criticisms, several authors show equilibrium outcomes with regard to foreclosure. I discuss such foreclosure problems in the latter part of the Introduction.

In the literature on vertical integration, however, there is no theoretical explanation why integration endogenously changes company strategies for product differentiation. In this paper, I develop an equilibrium theory of vertical merger that incorporates strategic behaviors in the Hotelling-type location model.

Our setting is as follows. There are two upstream firms and two downstream firms. Each is located in a linear city. Each downstream firm buys input from upstream firms. Each upstream firm engages in price competition for the business of downstream firms. The inputs produced by the upstream firms are perfect substitutes. To supply one unit of input for downstream firms, each upstream firm incurs transportation costs. I can interpret transportation costs as the loss of conversion from an upstream firm's product into a suitable input for a downstream firm. After purchasing input from an upstream firm, each downstream firm sets its retail price. Observing the retail prices, consumers choose to purchase from one of the downstream firms.<sup>3</sup>

This setting is suitable to explain vertical integration in the automobile and aircraft industries. For instance, in the aircraft industry, the jet (turboprop) engine and aircraft industries are vertically related. To produce a differentiated product, aircraft firms must procure suitable equipment. Bonaccorsi and Giuri (2001) reported the following: 'In the presence of economies of scope, engine programs are potentially applicable to different aircraft programs of different manufacturers. This allows engine companies to relate to many buyers, and potentially to all of them.' To explain these capabilities of upstream firms, some economists suppose that they are able to use a flexible manufacturing system (FMS). Eaton and Schmitt's (1994) framework of FMSs was explained as follows. 'By incurring a sunk cost of product development, upstream firms develop the ability to produce a *basic product*, described by a point in Hotelling's attribute space, at a constant marginal cost. A basic product can be modified to produce any other variant in the attribute space, but such mod-

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<sup>3</sup> This setting is based on that of Matsushima (2004). Using a restrictive setting in which upstream firms cannot choose their locations, Matsushima (2004) briefly considered vertical mergers. In this paper, I consider the location choice of upstream firms and discuss the mechanism of vertical merger(s) and the welfare implications of the vertical merger(s) in depth. This study provides new insight into the topic.

ification involves additional costs; the costs of switching the production process from one variant to another, and a per-unit cost of modification that is proportional to the difference between the basic product and the variant (1994, p. 875).’

Using the framework, I show that vertical integration enhances the degree of the product differentiation of the integrated firm. I also show that depending on the upstream firms’ transport costs, there are three patterns of vertical integration: no downstream firm vertically integrates, both downstream firms vertically integrate, and only one downstream firm vertically integrates (partial integration). In the third case, the profit earned by the non-integrated downstream firm is not always less than that in which no vertical integration appears. That is, vertical integration does not always harm the rival downstream firm.

I now show the intuition behind the first result. The increment in the degree of product differentiation has two effects. First, it increases the costs of the rival downstream firm. The procurement cost of each downstream firm depends on the distance between the downstream firm and its “potential” supplier (upstream firm) because of Bertrand competition between the upstream firms. To increase the procurement cost of the rival downstream firm, the integrated upstream firm tends to be far away from the rival downstream firm. Increasing the rival’s cost is beneficial for the integrated firm. Second, as pointed out in d’Aspremont *et al.* (1979), product differentiation mitigates the price competition between the downstream firms. The integrated firm does not take into account the procurement cost, which is related to the Bertrand competition between the upstream firms. The integrated downstream firm procures the input without costs. Therefore, the price effect is significant and the integrated firm tends to be far away from the rival firm.

The second result is related to several papers in which vertical integration and vertical foreclosure are considered (for instance, Choi and Yi (2000), Church and Gandal (2000), and references therein). As mentioned in Choi and Yi’s (2000) discussion of vertical foreclosure, the following question arises:<sup>4</sup> ‘Why will the nonintegrated downstream firm whose costs have been raised not react with a counter-merger of its own to mitigate the adverse effect of its

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<sup>4</sup> The statement is quoted from Choi and Yi (2000, p. 719). Bork (1978, p. 244) also raises the question.

rival's initial merger?' In Choi and Yi (2000), upstream firms can choose a specialized input for one particular downstream firm. The choice is effectively a commitment not to supply the other downstream firm. This induces the appearance of vertical foreclosure (partial integration). In Church and Gandal (2000), an integrated firm (a software and hardware firm) can make its software incompatible with a rival technology or system. The incompatibility is a driving force of vertical foreclosure (partial integration).

In this paper, a partial integration appears in equilibrium. The mechanism is different from those in the two papers cited above. Under the partial integration case, the profit of the non-integrated upstream firm increases as the transport cost of the upstream firms increases, because the upstream firm can set its wholesale price at a high level. This is a competitive disadvantage against the non-integrated downstream firm. To mitigate the disadvantage, the non-integrated downstream firm locates at a point near the center. The location increases the quantity demanded for the non-integrated downstream firm (positive) and price competition (negative). When the former (positive) effect dominates the latter (negative) one, the non-integrated downstream firm hesitates to integrate vertically. Moreover, as mentioned earlier, the integrated firm chooses its location at the edge. Maximum differentiation of the integrated firm mitigates the price competition and also enhances the profit of the non-integrated upstream firm. The mitigation is a disincentive to pursue full integration.

After discussion on the location choices and the integration decisions, I provide a welfare analysis. I show that partial integration does not improve social welfare, and suggest the reason. Through vertical integration, on the one hand, the integrated firm efficiently supplies its product. On the other hand, transportation costs paid by the consumers increase because the integrated firm fully differentiates its product. The former is positive and the latter is negative. In this setting, the latter is significant. Therefore, vertical integration does not improve social welfare. Moreover, the maximum differentiation of the integrated firm mitigates the price competition and then increases the prices of the downstream firms. This harms consumers. Therefore, I cannot say that vertical integration is pro-competitive or

competition neutral.

The remainder of this paper is organized as follows. Section 2 presents the basic model. Section 3 shows the location and pricing decisions. Section 4 shows the integration decisions. Section 5 provides some welfare implications. Section 6 concludes the paper. All proofs of lemmas and propositions are presented in the Appendix.

## 2 The model

There are two downstream firms,  $D_1$  and  $D_2$ , which produce the same physical product. There is a linear city of length 1, which lies on the abscissa of a line, and consumers are uniformly distributed with density 1 along the interval.<sup>5</sup> Suppose that  $D_1$  (*resp.*  $D_2$ ) is located at point  $l_1 \in [0, 1]$  (*resp.*  $1 - l_2 \in [0, 1]$ ). Without loss of generality, it is sufficient to consider only the case in which  $l_1 \leq 1 - l_2$ , that is, I label the left-hand- (*resp.* right-hand-) side firm as  $D_1$  (*resp.*  $D_2$ ). A consumer living at  $y \in [0, 1]$  incurs a transportation cost of  $t(l_1 - y)^2$  (*resp.*  $t(1 - l_2 - y)^2$ ) when purchasing a product from  $D_1$  (*resp.*  $D_2$ ). The consumers have unit demands, i.e., each consumes one or zero units of the product. Each consumer derives a surplus from consumption (gross of price and transportation costs) equal to  $s$ . I assume that  $s$  is so large that every consumer consumes one unit of the product. The above-mentioned assumptions are similar to those used by d'Aspremont *et al.* (1979).

Two upstream firms,  $U_A$  and  $U_B$ , supply inputs to two downstream firms. Suppose that  $U_A$  (*resp.*  $U_B$ ) is located at point  $h_A \in [0, 1]$  (*resp.*  $1 - h_B \in [0, 1]$ ). Without loss of generality, it is sufficient to consider only the case in which  $h_A \leq 1 - h_B$ , that is, I label the left-hand- (*resp.* right-hand-) side firm as  $U_A$  (*resp.*  $U_B$ ). Upstream firms engage in price competition for the business of downstream firms. Each input of a downstream firm produced by an upstream firm is a perfect substitute. To supply one unit of input for  $D_1$  and  $D_2$ ,  $U_A$  (*resp.*  $U_B$ ) incurs a cost of  $\tau(l_1 - h_A)^2$  and  $\tau(1 - l_2 - h_A)^2$  (*resp.*  $\tau(1 - h_B - l_1)^2$  and  $\tau(l_2 - h_B)^2$ ), where  $\tau$  is constant. For instance, if  $l_1 = h_A$ ,  $U_A$  can supply units of input for  $D_1$  without costs.<sup>6</sup> I assume that  $\tau \leq t$ .

<sup>5</sup> This formulation is originally due to Hotelling (1929) and d'Aspremont *et al.* (1979).

<sup>6</sup> I have assumed that only two upstream firms exist. Suppose that a potential upstream firm has to incur

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Figure 1

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I interpret the distance between  $D_i$  and  $U_j$  as (1) literally “distance” or (2) the loss of conversion from  $U_j$ 's product into the suitable input for  $D_i$ . The second interpretation is similar to those in Eaton and Schmitt (1994), Norman and Thisse (1999), and Belleflamme and Toulemonde (2003), in which a flexible manufacturing system is discussed.<sup>7</sup> Downstream firms transform one unit of the intermediate good into one unit of the final good, and they are assumed to incur no costs other than the input prices they pay to the upstream firms.

In this model, because of notational simplicity, upstream and downstream firms locate on the same line. As mentioned earlier, I interpret the location of upstream firms as the loss of conversion from  $U_j$ 's product into a suitable input for  $D_i$ . That is, the linear city should be interpreted as a technology space when I evaluate the location of upstream firms.

To develop an equilibrium theory of vertical integration, I follow the method of Pepall *et al.* (2004, p. 439). I analyze a stage game. In the first stage, upstream and downstream firms decide simultaneously whether to integrate vertically. If a vertical merger takes place I assume that  $D_1$  ( $D_2$ ) merges with  $U_A$  ( $U_B$ ). In the second stage, downstream firms and upstream firms simultaneously choose their locations. In the third stage, each upstream firm,  $U_i$  ( $i = A, B$ ), simultaneously chooses its wholesale prices,  $w_{ij} \in [0, \infty)$  ( $j = 1, 2$ ), where  $j$  is the index of the downstream firm,  $D_j$  ( $j = 1, 2$ ). For instance,  $w_{A2}$  is  $U_A$ 's wholesale price for  $D_2$ . The upstream firms compete using spatial discriminatory pricing. In the fourth stage, when observing the wholesale prices, each downstream firm chooses between  $U_A$  and  $U_B$  as its supplier and then sets its retail price  $p_i \in [0, \infty)$  ( $i = 1, 2$ ) simultaneously. In the fifth stage, by observing the retail prices, consumers choose to purchase from either  $D_1$  or  $D_2$ . A sunk cost of product development when it enters the market. If a third (potential) upstream firm really enters the market, one of the upstream firms faces a fierce Bertrand competition and can earn no profit in equilibrium because only two downstream firms exist and upstream firms engage in price competition.

<sup>7</sup> I have mentioned that the setting is somewhat similar to a spatial price discrimination model. For a discussion of the spatial price discrimination model, see Lederer and Hurter (1986), MacLeod *et al.* (1988), and Hamilton *et al.* (1989).



$D_2$ .

For this paper, I only consider the case in which  $l_1 \leq 1 - l_2$ . For a consumer living at:

$$(1) \quad x = \frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)},$$

the total cost (transportation cost plus price) is the same from either of the two firms.<sup>8</sup>

### 3 Location and pricing decisions

Before I consider the decisions of vertical integration, I discuss three separate cases: no integration, partial integration (one vertical integration occurs), and full integration (two vertical integrations occur). To discuss the three cases, I provide analyses of the second to the fifth stages for each case.

#### 3.1 No integration

I first discuss a case in which no integration occurs, which was also discussed in Matsushima (2004). I provide this case as a benchmark.

The profit of each downstream firm is given by:

$$(2) \quad \pi_{d1} \equiv (p_1 - \min\{w_{A1}, w_{B1}\}) \left( \frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)} \right),$$

$$(3) \quad \pi_{d2} \equiv (p_2 - \min\{w_{A2}, w_{B2}\}) \left( \frac{1 - l_1 + l_2}{2} + \frac{p_1 - p_2}{2t(1 - l_1 - l_2)} \right).$$

In (2) and (3),  $\min\{w_{A1}, w_{B1}\}$  (*resp.*  $\min\{w_{A2}, w_{B2}\}$ ) means that  $D_1$  (*resp.*  $D_2$ ) procures its input from the upstream firm ( $U_A$  or  $U_B$ ) that bids the lowest wholesale price.

Calculating the stage game, the following proposition is derived:

**Proposition 1** (*Matsushima (2004)*) *In the no-integration case, the following location pat-*

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<sup>8</sup> Equation (1) is derived by the following equation:  $t(x - l_1)^2 + p_1 = t(1 - l_2 - x)^2 + p_2$ .

tern is an equilibrium outcome:

$$\begin{aligned}
(4) \quad & \text{if } \tau \leq \frac{t}{4}, & l_1 = l_2 = 0, & h_A = h_B = 0, \\
& \text{if } \frac{t}{4} < \tau \leq \frac{3t}{7}, & l_1 = l_2 = \frac{4\tau - t}{4(t + \tau)}, & h_A = h_B = 0, \\
& \text{if } \frac{3t}{7} < \tau, & l_1 = l_2 = \frac{4t + 13\tau - 3\sqrt{4t^2 + 4t\tau + 9\tau^2}}{8(t + \tau)}, \\
& & h_A = h_B = \frac{3\sqrt{4t^2 + 4t\tau + 9\tau^2} - (6t + 5\tau)}{8\tau}.
\end{aligned}$$

In the equilibrium outcome, the profits of each firm and the prices of each firm are:

$$\begin{aligned}
(5) \quad & \text{if } \tau \leq \frac{t}{4}, & \pi_{d1} = \pi_{d2} = \frac{t}{2}, & \pi_{u1} = \pi_{u2} = \frac{\tau}{2}, & p_1 = p_2 = t + \tau, \\
& \text{if } \frac{t}{4} < \tau \leq \frac{3t}{7}, & \pi_{d1} = \pi_{d2} = \frac{t(3t - 2\tau)}{4(t + \tau)}, & \pi_{u1} = \pi_{u2} = \frac{\tau(3t - 2\tau)}{4(t + \tau)}, \\
& & p_1 = p_2 = \frac{t(24t^2 + 33t\tau - 16\tau^2)}{16(t + \tau)^2}, \\
& \text{if } \frac{3t}{7} < \tau, & \pi_{d1} = \pi_{d2} = \frac{3t(\sqrt{4t^2 + 4t\tau + 9\tau^2} - 3\tau)}{8(t + \tau)}, \\
& & \pi_{u1} = \pi_{u2} = \frac{9(\sqrt{4t^2 + 4t\tau + 9\tau^2} - 3\tau)}{32(t + \tau)} \\
& & \quad \times (2t + 3\tau - \sqrt{4t^2 + 4t\tau + 9\tau^2}), \\
& & p_1 = p_2 = \frac{3t[3(2t - \tau)(2t^2 + 7t\tau + 8\tau^2)]}{32\tau(t + \tau)^2} \\
& & \quad - \frac{3t[(6t^2 + 7t\tau - 8\tau^2)\sqrt{4t^2 + 4t\tau + 9\tau^2}]}{32\tau(t + \tau)^2}.
\end{aligned}$$

To understand the benchmark case, I now show the intuition behind Proposition 1.<sup>9</sup> In this model, given the locations of the other three firms, when a downstream firm moves farther away from its rival, three effects occur. First, price competition between downstream firms is softened (“price effect”). The price effect, which is similar to that in d’Aspremont *et al.* (1979), enhances the profit of the downstream firm ( $D_1$ ). Second, the demand for the downstream firm falls (“demand effect”). The demand effect, which is similar to that in d’Aspremont *et al.* (1979), diminishes the profit of the downstream firm ( $D_1$ ). Third, the wholesale price for the downstream firm rises (“input price effect”) because the distance between the downstream firm and the other supplier increases. (e.g.,  $D_1$  and  $U_B$ ).

<sup>9</sup> The intuition mentioned in the paper is also mentioned in Matsushima (2004).

The input price effect is affected by the value of  $\tau$ . The larger  $\tau$  is, the larger are the wholesale prices. On the other hand, the price and demand effects are unaffected by the value of  $\tau$  because the cost of each downstream firm is equal to that of its rival in equilibrium. Therefore, when  $\tau$  is large, the third effect (input price effect) is important for downstream firms. To lower the input price, each downstream firm shortens the distance to the rival's supplier, and the maximal differentiation result does not hold. If  $\tau > t/4$ , given the location pattern in (4), the larger the value of  $\tau$  is, the smaller the profit of each downstream firm.

### 3.2 Partial integration

In this case, a pair of an upstream and a downstream firm exists. As pointed out by Reiffen (1992), I have assumed that the integrated firm ( $U_B$  and  $D_2$ ) cannot commit to not supply units of input to the rival downstream firm ( $D_1$ ).

#### 3.2.1 The fourth and fifth stages

For a consumer living at  $x$  in (1), the total cost is the same at either of the two firms. Each downstream firm's profit is given by:

$$(6) \quad \pi_{d1} \equiv (p_1 - \min\{w_{A1}, w_{B1}\}) \left( \frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)} \right),$$

$$(7) \quad \pi_I \equiv (p_2 - \tau(h_B - l_2)^2) \left( \frac{1 - l_1 + l_2}{2} + \frac{p_1 - p_2}{2t(1 - l_1 - l_2)} \right).$$

In (7), I consider a case in which the integrated downstream firm procures its input from its own upstream firm and incurs a transportation cost,  $\tau(h_B - l_2)^2$ . In this case, the procurement cost of the integrated firm is not related to the location of  $U_A$ . Considering the following two cases, I shall explain the procurement cost. First, if the distance between  $U_A$  and  $D_2$  is shorter than that between  $U_B$  and  $D_2$ ,  $U_A$  supplies  $D_2$  and sets the wholesale price at the transportation cost of  $U_B$  (that is,  $\tau(h_B - l_2)^2$ ). Second, if the distance between  $U_A$  and  $D_2$  is further than the distance between  $U_B$  and  $D_2$ ,  $U_B$  supplies  $D_2$  and sets the wholesale price at the transportation cost of  $U_B$  (that is,  $\tau(h_B - l_2)^2$ ) because  $U_B$  and  $D_2$  are integrated. For any locations of  $U_A$  and  $U_B$ , the procurement cost of  $D_2$  is  $\tau(h_B - l_2)^2$ .

Calculating the first-order conditions of downstream firms, I have:

$$\begin{aligned}\pi_{d1} &= \frac{((1 - l_1 - l_2)(3 + l_1 - l_2)t - \min\{w_{A1}, w_{B1}\} + \tau(h_B - l_2)^2)^2}{18(1 - l_1 - l_2)t}, \\ \pi_I &= \frac{((1 - l_1 - l_2)(3 - l_1 + l_2)t + \min\{w_{A1}, w_{B1}\} - \tau(h_B - l_2)^2)^2}{18(1 - l_1 - l_2)t}.\end{aligned}$$

### 3.2.2 The third stage

For  $D_1$  (a non-integrated downstream firm),  $U_A$  (a non-integrated upstream firm) and the integrated firm engage in price competition. In this case,  $U_A$  supplies to  $D_1$  and sets the wholesale price at  $w_{A1} = \tau(1 - h_B - l_1)^2$ , which is the transportation cost of the integrated upstream firm when it supplies to  $D_1$ .<sup>10</sup> To supply for  $D_1$ ,  $U_A$  incurs a per unit transportation cost,  $\tau(l_1 - h_A)^2$ . From the results in Section 3.2.1, I can derive the profit functions of the rival downstream and upstream firms and the integrated firm:

$$(8) \quad \pi_{d1} = 2t(1 - l_1 - l_2)X^2,$$

$$(9) \quad \pi_{u1} = [\tau(1 - h_B - l_1)^2 - \tau(l_1 - h_A)^2]X,$$

$$(10) \quad \pi_I = 2t(1 - l_1 - l_2)(1 - X)^2,$$

where  $X \equiv \frac{(3 + l_1 - l_2)t - (1 - h_B - l_1)\tau - (l_2 - h_B)\tau}{6t}$ .

$X$  is the quantity supplied by  $D_1$  and  $1 - X$  is the quantity supplied by  $I$ .

<sup>10</sup> If  $\tau$  is sufficiently large, the “monopoly” price set by  $U_A$  may be smaller than the rival ( $U_B$ )’s wholesale price (which is equal to the rival’s transportation cost per unit). To explain the pricing strategy of  $U_A$ , I now assume that  $U_B$  is unable to supply input to  $D_1$ . Under the assumption,  $U_A$  is the monopolist of the input for  $D_1$ .  $U_A$  takes into account the relation between its wholesale price  $w_{A1}$  and the quantity supplied by  $D_1$ , that is,  $U_A$  faces the derived demand for its input.  $U_A$  sets its wholesale price to maximize its profit. When the “monopoly” price is lower than  $U_B$ ’s transportation cost per unit (this is related to  $\tau$ ), even though  $U_B$  is able to supply its input to  $D_1$ ,  $U_B$  has no incentive to meet  $D_1$ ’s demand for input under the monopoly pricing of  $U_A$ . In the model, given the equilibrium locations, the “monopoly” price is always higher than the per unit transportation cost of the rival upstream firm. Therefore, for locations, I do not have to consider monopoly pricing. However, when I discuss any location pattern, I have to take into account the “monopoly” pricing. The discussion of the “monopoly” pricing by  $U_A$  complicates the analysis. Because vertical integrations appear in equilibrium for a lower value of  $\tau$ , I suggest that considering the “monopoly” pricing of  $U_A$  may not provide additional insight into the discussion on vertical integrations. Moreover, if  $\tau$  is large enough, the pure strategy location equilibrium does not exist in the partial integration case. The assumption that  $\tau \leq t$  avoids the non-existence problem. In this paper, therefore, I only consider the case in which  $\tau \leq t$ . In this case, the problems do not occur.

### 3.2.3 The second stage

Calculating the first-order conditions, I have:

$$(11) \quad \frac{\partial \pi_{d1}}{\partial l_1} = -\frac{(t(1 + 3l_1 + l_2) - (3 - 3l_1 - l_2 - 2h_B)\tau)X}{3},$$

$$(12) \quad \frac{\partial \pi_{u1}}{\partial h_A} = 2\tau(l_1 - h_A)X,$$

$$(13) \quad \frac{\partial \pi_I}{\partial l_2} = -\frac{(t(1 + l_1 + 3l_2) - (1 - l_1 - 3l_2 + 2h_B)\tau)(1 - X)}{3},$$

$$(14) \quad \frac{\partial \pi_I}{\partial h_B} = -\frac{4(1 - l_1 - l_2)\tau X}{3}.$$

In the paper, I only focus on an interior solution of locations, and  $X$  is a positive value. In the Appendix, I show that the derived interior solution in this section is an equilibrium outcome.

From (12),  $U_1$  chooses its location to satisfy  $\partial \pi_{u1}/\partial h_A = 0$ , that is,  $h_A = l_1$ . I shall mention the reason that  $h_A = l_1$ . The integrated downstream firm procures its input from its own upstream firm. The procurement cost of the integrated firm is not related to the location of  $U_A$  (the rival upstream firm). If the distance between  $U_A$  and  $D_1$  is shorter than that between  $U_B$  and  $D_1$ , the wholesale price of  $D_1$ ,  $w_{A1} = \tau(1 - h_B - l_1)^2$ , is not related to the location of  $U_A$  either. Moreover, the quantity supplied by  $U_A$  is not related to the location of  $U_A$  because the input prices ( $w_{A1}$  and  $w_{B2}$ ) are not related to the location of  $U_A$ . Therefore,  $U_A$ 's purpose is to minimize its transportation cost. To reduce the transportation cost,  $U_A$  chooses a location at the same point as  $D_1$ , that is,  $h_A = l_1$ .

From (14),  $\partial \pi_I/\partial h_B < 0$ , unless  $l_1 = 1 - l_2$  (note that, without loss of generality, I have only considered the case in which  $l_1 \leq 1 - l_2$ ). Therefore,  $h_B = 0$ . I shall demonstrate the reason that  $h_B = 0$ . To raise the procurement cost of the rival downstream firm  $D_1$ ,  $U_B$  tends to be far away from  $D_1$ . Because of the convexity of transport technology, the increment of  $D_1$ 's procurement cost is larger than that for  $D_2$ .<sup>11</sup> This raises the relative level of the rival's cost and is beneficial for the integrated firm. The best way to increase the rival's cost is when the integrated upstream firm locates at the furthest point from the rival

<sup>11</sup> If  $D_1$  and  $D_2$  locate at the same point (that is,  $l_1 = 1 - l_2$ ), the effect disappears. The condition is reflected in (14).

downstream firm, that is,  $h_B = 0$ .

Substituting  $h_A = l_1$  and  $h_B = 0$  into (11) and (13), I have the following simultaneous equations:

$$(11') \quad -\frac{(t(1 + 3l_1 + l_2) - (3 - 3l_1 - l_2 - 2h_B)\tau)X}{3} = 0,$$

$$(13') \quad -\frac{(t(1 + l_1 + 3l_2) - (1 - l_1 - 3l_2 + 2h_B)\tau)(1 - X)}{3} = 0.$$

Solving the equations, I have the following proposition:<sup>12</sup>

**Proposition 2** *In the partial integration case, the following location pattern is an equilibrium outcome:*

(i) if  $\tau \leq t/3$ ,

$$(17) \quad \begin{aligned} l_1 = h_A = 0, \quad l_2 = h_B = 0, \quad w_{A1} = \tau, \\ p_1 = \frac{3t + 2\tau}{3}, \quad p_2 = \frac{3t + \tau}{3}, \\ \pi_{d1} = \frac{(3t - \tau)^2}{18t}, \quad \pi_{u1} = \frac{(3t - \tau)\tau}{6t}, \quad \pi_I = \frac{(3t + \tau)^2}{18t}, \quad x = \frac{3t - \tau}{6t}. \end{aligned}$$

(ii) if  $\tau > t/3$ ,

$$(18) \quad \begin{aligned} l_1 = h_A = \frac{3\tau - t}{3(t + \tau)}, \quad l_2 = h_B = 0, \quad w_{A1} = \frac{16t^2\tau}{9(t + \tau)^2}, \\ p_1 = \frac{16(2t + 5\tau)t^2}{27(t + \tau)^2}, \quad p_2 = \frac{40t^2}{27(t + \tau)}, \\ \pi_{d1} = \frac{128t^2}{243(t + \tau)}, \quad \pi_{u1} = \frac{64t^2\tau}{81(t + \tau)^2}, \quad \pi_I = \frac{200t^2}{243(t + \tau)}, \quad x = \frac{4}{9}. \end{aligned}$$

In the partial integration case,  $l_1 = h_A$  and  $l_2 = h_B = 0$  for any  $\tau (< t)$ . That is, as mentioned in the Introduction, vertical integration enables the integrated firm to make more differentiated products.

When  $\tau$  is large,  $D_1$  does not locate at the edge. The reason is similar to that in the case of no integration. The input price effect is important for  $D_1$ . To reduce its procurement cost,  $D_1$  tends to locate near  $U_2$ . The equilibrium location of  $D_1$  is superior to locating at the edge from the viewpoint of social surplus.

I shall mention the reason that  $l_2 = 0$ . As stated earlier, for any locations of  $U_A$  and  $U_B$ , the procurement cost of  $D_2$  is  $\tau(h_B - l_2)^2$ . In that case, the input price effect of  $D_2$  does

<sup>12</sup> The way to derive the proposition is presented in the Appendix.

not exist, and the integrated upstream firm locates at  $h_B = 0$ . Moreover, as pointed out in d'Aspremont *et al.* (1979), the price effect is also important and  $D_2$  tends to be far away from the rival firm. Locating at  $l_2 = 0$  maximizes the distance between the downstream firms and minimizes the distance between the integrated upstream and downstream firms (as a result,  $\tau(h_B - l_2)^2 = 0$ ).

### 3.3 Full integration

In this case, two pairs exist, with each being made up of an upstream and a downstream firm.

#### 3.3.1 The third, fourth, and fifth stages

For a consumer living at  $x$  in (1), the total cost is the same at either of the two firms. Each downstream firm's profit is given by:

$$(19) \quad \pi_{I1} \equiv (p_1 - \tau(h_A - l_1)^2) \left( \frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)} \right),$$

$$(20) \quad \pi_{I2} \equiv (p_2 - \tau(h_B - l_2)^2) \left( \frac{1 - l_1 + l_2}{2} + \frac{p_1 - p_2}{2t(1 - l_1 - l_2)} \right).$$

As mentioned in the previous subsection, in (19) and (20), I consider the case in which each integrated downstream firm procures its input from its own upstream firm and incurs its transportation cost,  $\tau(h_A - l_1)^2$  ( $\tau(h_B - l_2)^2$ ). In this case, the procurement cost of the integrated firm is not related to the location of the rival's upstream firm. For any locations of  $U_A$  and  $U_B$ , the procurement cost of  $D_1$  ( $D_2$ ) is  $\tau(h_A - l_1)^2$  ( $\tau(h_B - l_2)^2$ ). Calculating the first-order conditions of downstream firms, I have:

$$(21) \quad \pi_{I1} = \frac{((1 - l_1 - l_2)(3 + l_1 - l_2)t - \tau(h_A - l_1)^2 + \tau(h_B - l_2)^2)^2}{18(1 - l_1 - l_2)t},$$

$$(22) \quad \pi_{I2} = \frac{((1 - l_1 - l_2)(3 - l_1 + l_2)t + \tau(h_A - l_1)^2 - \tau(h_B - l_2)^2)^2}{18(1 - l_1 - l_2)t}.$$

#### 3.3.2 The second stage

I consider the location choices of the integrated firms. From (21) and (22), the first-order conditions are:

$$(23) \quad \frac{\partial \pi_{I1}}{\partial l_1} = - \frac{((1 + 3l_1 + l_2)(1 - l_1 - l_2)t + (l_1 - h_A)(4 - h_A - 3l_1 - 4l_2)\tau - (l_2 - h_B)^2\tau)Y}{3(1 - l_1 - l_2)},$$

$$(24) \quad \frac{\partial \pi_{I1}}{\partial h_A} = \frac{4(l_1 - h_A)\tau Y}{3},$$

$$(25) \quad \frac{\partial \pi_{I2}}{\partial l_2} = - \frac{((1 + l_1 + 3l_2)(1 - l_1 - l_2)t + (l_2 - h_B)(4 - h_B - 3l_2 - 4l_1)\tau - (l_1 - h_A)^2\tau)(1 - Y)}{3(1 - l_1 - l_2)},$$

$$(26) \quad \frac{\partial \pi_{I2}}{\partial h_B} = \frac{4(l_2 - h_B)\tau(1 - Y)}{3},$$

$$\text{where } Y \equiv \frac{(1 - l_1 - l_2)(3 + l_1 - l_2)t - \tau(h_A - l_1)^2 + \tau(h_B - l_2)^2}{6(1 - l_1 - l_2)t}.$$

$Y$  is the quantity supplied by  $D_1$  and  $1 - Y$  is the quantity supplied by  $I$ .  $Y$  is positive. From (24) and (26), I find that  $h_A = l_1$  and  $h_B = l_2$ . Substituting these into (23) and (25), I have that  $\partial \pi_{Ij} / \partial l_j < 0$  ( $j = 1, 2$ ) for any  $l_1 \leq 1 - l_2$ . Therefore, the following result holds:

$$(27) \quad l_1 = h_A = 0, \quad l_2 = h_B = 0, \quad p_1 = p_2 = t, \quad \pi_{I1} = \pi_{I2} = \frac{t}{2}, \quad x = \frac{1}{2}.$$

The case is similar to that in d'Aspremont *et al.* (1979); the marginal cost of each (downstream) firm is zero but does not depend on the locations of the firms. Therefore, the maximum differentiation appears in equilibrium.

## 4 Integration decisions

I shall discuss decisions regarding vertical integration; that is, the first stage of the game is discussed. The first stage is represented by the following  $2 \times 2$  matrix.<sup>13</sup>

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Table 1

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$\Pi_N$ ,  $\Pi_i$ ,  $\Pi_n$ , and  $\Pi_I$  are derived by the following calculations:

$$\begin{aligned} \Pi_N: & \quad \pi_{d1} + \pi_{u1} \text{ in (5);} \\ \Pi_n: & \quad \pi_{d1} + \pi_{u1} \text{ in (17) if } \tau \leq t/3, \text{ otherwise, } \pi_{d1} + \pi_{u1} \text{ in (18);} \\ \Pi_i: & \quad \pi_I \text{ in (17) if } \tau \leq t/3, \text{ otherwise, } \pi_I \text{ in (18);} \\ \Pi_I: & \quad \pi_I \text{ in (27).} \end{aligned}$$

<sup>13</sup> This representation method is similar to that in Pepall *et al.* (2004, p. 442).



Calculating the difference between  $\Pi_I$  and  $\Pi_n$ , I derive the following lemma:

**Lemma 1** *The firms' profits when full integration occurs are smaller than the sum of an upstream and a downstream firm's profits when partial integration occurs, if and only if  $\tau < \frac{(77+8\sqrt{142})t}{243} \sim 0.709t$ .*

Lemma 1 states that the non-integrated firms do not react with a counter-merger of their own unless the transport costs of the upstream firm are large enough. The result may be surprising because each firm is symmetric and no specific assumption about the pricing strategy of an integrated upstream firm exists.

I shall now demonstrate the mechanism behind Lemma 1. In the partial integration case, the profit of the non-integrated upstream firm increases with the value of  $\tau$  because the upstream firm can set its wholesale price at a high level. This is a competitive disadvantage against the non-integrated downstream firm. To mitigate this disadvantage, it locates at a point near the center (the input price effect). The location is advantageous from the viewpoint of the demand effect but not from the viewpoint of the price effect (see Section 3.1). The demand effect reduces the incentive of the non-integrated downstream firm to integrate vertically. However, as the value of  $\tau$  increases, the input price effect is more significant, and the price competition between the downstream firms then intensifies because the distance between them  $(1 - l_1 - l_2)$  shortens (see (18)). Therefore, when the value of  $\tau$  is large enough, the non-integrated downstream firm attempts to merge to avoid the tougher price competition.

Before I discuss the differences among the related papers and my model, I consider the effect of vertical integration on the input prices. With no integration, the input prices of the downstream firms are  $\tau(1 - h_B - l_1)^2$  and  $\tau(1 - l_2 - h_A)^2$ . Using (4), I can calculate the values. The input price under partial integration is mentioned in (17) and (18). After some calculus, I have the following Proposition:

**Proposition 3** *If  $\tau \leq t/4$ , the input price of the non-integrated downstream firm under partial integration is as high as that without integration. If  $\tau > t/4$ , the input price of*

*the non-integrated downstream firm under partial integration is larger than that with no integration.*

As mentioned earlier, the integrated upstream firm locates at the edge. This serves as a commitment to increase the input price facing the rival downstream firm. From Proposition 3, I find that the effect of the increased cost to the rival also exists in the model.

The driving force of a partial integration is different from those in Choi and Yi (2000) and Church and Gandal (2000). In both those papers, an integrated firm specializes its input for its downstream component. The inputs are unsuitable for its rival downstream firm. The specialization is a commitment device not to supply the rival. In my model, an upstream firm's location can be interpreted as its choice of input specialization. Further, the fact that this choice is made before pricing occurs means that it serves as a commitment to raise the input price facing the rival downstream firm. The commitment is somewhat similar to that discussed in the aforementioned papers and increases the profit of the non-integrated upstream firm. As mentioned earlier, however, the integrated firm chooses its location at the edge on the Hotelling line. The endogenous product differentiation is an important factor in my paper. The maximum differentiation of the integrated firm mitigates the price competition and then enhances the profit of the non-integrated upstream firm. The mitigation is a disincentive to full integration.

To show the condition under which partial integration occurs, I calculate the difference between  $\Pi_i$  and  $\Pi_N$ , and I have the following lemma:

**Lemma 2** *The integrated firm's profit in which partial integration occurs is smaller than the sum of an upstream and a downstream firms' profits in which no integration occurs if and only if  $\tau < \frac{3(3\sqrt{3}-5)t}{2} \simeq 0.294t$ .*

When the value of  $\tau$  is large, price competition is severe in the no integration case because each downstream firm tends to access each potential upstream firm to reduce their procurement costs. A vertical integration mitigates price competition because the integrated firm does not have to access the potential supplier. This induces vertical integration.

From Lemmas 1 and 2, I have the following propositions.

**Proposition 4** *When vertical integration is determined endogenously, no integration occurs if and only if  $\tau < \frac{3(3\sqrt{3}-5)t}{2}$  ( $\simeq 0.294t$ ), partial integration occurs if and only if  $\frac{3(3\sqrt{3}-5)t}{2} \leq \tau < \frac{(77+8\sqrt{142})t}{243}$  ( $\simeq 0.709t$ ), and full integration occurs if and only if  $\frac{(77+8\sqrt{142})t}{243} \leq \tau$ .*

The non-integrated downstream firm may also benefit from the vertical integration because its location is more advantageous than that of the integrated firm (see  $l_1$  and  $l_2$  in (18)). Calculating the difference between  $\Pi_n$  and  $\Pi_N$ , I derive the following proposition:

**Proposition 5** *The profit of the non-integrated downstream firm in which a partial integration occurs is larger than that in which no integration occurs if and only if  $\tau > \frac{269297t}{588303} \simeq 0.458t$ .*

The integration mitigates price competition between the downstream firms. However, the non-integrated downstream firm is less efficient than the integrated firm. In the Hotelling model, the inefficiency is less serious for the non-integrated downstream firm because of strategic complementarity. When  $\tau$  is large, because of the input price effect, the price competition that stems from the proximity between them is more significant. Therefore, Proposition 5 holds.

## 5 Welfare analysis

I now briefly discuss social welfare and consumer surplus. In this model, the only source of inefficiency is the transportation costs paid by the consumers and by the upstream firms, due to the fact that the market is fully covered.<sup>14</sup> Social welfare and consumer surplus are expressed by the following equations ( $x^*$  is the indifferent consumer):

$$(28) \quad SW = s - \left( \int_0^{x^*} t(m - l_1)^2 dm + \int_{x^*}^1 t(1 - l_2 - m)^2 dm \right) - \left( x^* \tau (l_1 - h_A)^2 + (1 - x^*) \tau (l_2 - h_B)^2 \right),$$

$$(29) \quad CS = s - \left( \int_0^{x^*} t(m - l_1)^2 dm + \int_{x^*}^1 t(1 - l_2 - m)^2 dm \right) - (x^* p_1 + (1 - x^*) p_2).$$

<sup>14</sup> In this model, because of the inelastic demand structure, high downstream prices do not entail deadweight loss, which is the case in many models with elastic demand structure.

In (28) and (29), the terms in the first parentheses are the transportation costs paid by the consumers. In (28), the terms in the second parentheses are the transportation costs paid by the upstream firms. In (29), the terms in the second parentheses are the prices paid by the consumers. Before I proceed with the welfare analysis, recall that the first-best location pattern is when  $l_1 = l_2 = 1/4$  and  $h_A = h_B = 1/4$ . As is known from the literature of spatial competition, the location pattern,  $l_1 = l_2 = 1/4$ , minimizes the transportation costs paid by the consumers, given that the downstream firms' prices are the same. When an upstream firm locates at the same point as a downstream firm (that is,  $h_A = l_1$  and  $h_B = l_2$ ), the transportation costs paid by the upstream firm are zero.

I shall now discuss social welfare, and define three labels:  $SW_F$  is social welfare when full integration occurs;  $SW_P$  is social welfare when partial integration occurs; and  $SW_N$  is social welfare when no integration occurs. I derive the following proposition:

**Proposition 6** *For  $\tau < t$ , the following inequalities hold:*

$$\begin{aligned}
(30) \quad & SW_N = SW_F > SW_P, && \text{if and only if } \tau \leq \frac{t}{4}, \\
& SW_N > SW_F > (=) SW_P, && \text{if and only if } \frac{t}{4} < \tau < (=) \frac{(205-16\sqrt{55})t}{243} \simeq 0.355t, \\
& SW_N > SW_P > SW_F, && \text{if and only if } \frac{(205-16\sqrt{55})t}{243} < \tau.
\end{aligned}$$

I shall now demonstrate the intuition behind Proposition 6. As mentioned earlier, the transportation costs paid by the consumers and by the upstream firms induce welfare loss. The transportation costs paid by the consumers (*resp.* the upstream firms) depend on the market share related to the downstream prices and the locations of the downstream firms (*resp.* the upstream and downstream firms). The market share effect means that, given the locations of the downstream firms, separating the market at the mid-point between them is optimal and the optimal division is achieved when the downstream prices are equal.

I separate the discussion into two parts: (i)  $\tau \leq t/4$ ; (ii)  $\tau > t/4$ .

When  $\tau \leq t/4$ , the location patterns in the three cases are the same. Under partial integration, the distortion of the market share exists because the difference between the procurement costs of the downstream firms exists and this induces the difference of the downstream prices. Therefore,  $SW_P$  is the smallest.

When  $\tau > t/4$ , the differences among the location patterns of the three cases appear. In the non-integrated case, there is no distortion in the market share. The equilibrium locations of the downstream firms are the most efficient among the three cases because each downstream firm tends to access each potential upstream firm (the input price effect). The consumers' transportation costs are the smallest among the three cases. The transportation costs of the upstream firms are positive, but in the other two cases are zero. The inefficiency stemming from the upstream firms' transportation costs is not significant because the distances between the trading partners are short. Therefore, social welfare in the non-integration case is the highest of the three.

I shall now discuss the remaining two cases: partial integration and full integration. As mentioned earlier, the transportation costs of the upstream firms are zero in both cases. I can only focus on the transportation costs of the consumers. There is no distortion of the market share under full integration, but there is under partial integration. The equilibrium location pattern, however, is less efficient in the case of full integration if  $\tau > t/3$ . As the value of  $\tau$  increases, because of the input price effect, the efficiency of the equilibrium location pattern improves under partial integration. Therefore, Proposition 6 holds.

In the model, vertical integration causes social welfare to deteriorate. As mentioned earlier, vertical integration enables the integrated firm to make a more differentiated product. The locations of the integrated upstream and downstream firms are at the edge of the Hotelling line. This reduces social welfare because the transportation cost paid by the consumers increases. This is the main factor that makes full integration less efficient. Note that a vertical integration does not increase the volume of consumption in the model, because of the inelastic demand structure. Ordinarily, a vertical integration solves the problem of double marginalization, also increasing the volume of consumption and enhancing social welfare. Therefore, the reduction of social welfare under vertical integration may be overestimated. To discuss the relation between vertical integrations and social welfare accurately, I think that the relation between vertical integration and downstream prices should be discussed. I discuss this after Proposition 7.

Finally, I shall briefly discuss consumer surplus. I now define three labels:  $CS_F$  is consumer surplus when full integration occurs;  $CS_P$  is consumer surplus when partial integration occurs; and  $CS_N$  is consumer surplus when no integration occurs. I derive the following proposition:

**Proposition 7** *For  $\tau < t$ , the following inequalities hold:*

$$(31) \quad \begin{aligned} CS_F > CS_P > (=) CS_N, & \text{ if and only if } \tau < (=) \bar{\tau} \simeq 0.320t, \\ CS_F > (=) CS_N > CS_P, & \text{ if and only if } \bar{\tau} < \tau < (=) \frac{(25\sqrt{1417}-779)t}{352} \simeq 0.460t, \\ CS_N > CS_F > (=) CS_P, & \text{ if and only if } \frac{(25\sqrt{1417}-779)t}{352} < \tau < (=) \frac{455t}{729} \simeq 0.624t, \\ CS_N > CS_P > CS_F, & \text{ if and only if } \frac{455t}{729} < \tau, \end{aligned}$$

where  $\bar{\tau}$  satisfies the following equation:

$$\frac{5t(17t - 4\tau)}{48(t + \tau)} = \frac{39t^2 + 18t\tau - \tau^2}{36t}.$$

From Propositions 4 and 7, I find that the equilibrium integration pattern is never the best one from the viewpoint of consumer surplus.

I shall now show the intuition behind Proposition 7. As expressed in (29), the transportation costs and the prices paid by consumers reduce consumer surplus. The transportation costs paid by the consumers depend on the market share related to the downstream prices and the locations of the downstream firms, an effect already discussed. The prices paid by the consumers depend on the locations of the downstream firms and the procurement costs of the downstream firms. These costs are monetary transfers from the downstream firms to the upstream firms and are independent of social surplus.

I consider the procurement costs. Each downstream firm shifts its procurement cost to its price. These shifts harm the consumers. As mentioned earlier, an integrated firm does not incur the cost of its input, but a non-integrated downstream firm has to incur the input cost. Judged on that factor, full integration is the most efficient organizational structure, and non-integration is the worst. The first inequalities in Proposition 7 reflect the effect of the procurement costs.

The degree of each factor depends on the value of  $\tau$ . On the one hand, as the value of  $\tau$  increases, the procurement costs of non-integrated downstream firm(s) increase. On

the other hand, because of the input price effect, as the value of  $\tau$  increases, the difference among the location patterns under the three cases becomes significant. No integration is the most efficient and full integration is the worst. The location patterns affect not only the transportation costs paid by the consumers but also the downstream prices. The shorter the distance between the downstream firms, the tougher the price competition between them. In this model, the input price effect is more significant than the procurement costs, which are related to the non-integrated downstream firm(s). Therefore, Proposition 7 holds.

I shall discuss the relation between vertical integrations and the downstream prices. From  $p_1$  and  $p_2$  in (5), (17), (18), and (27), I depict the relation between the downstream prices and the level of  $\tau$  in the three cases (non, partial, and full integration).

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Figure 2

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$p_N$  is the price(s) in the non-integrated case;  $p_n$  is the price of the non-integrated downstream firm in the partially integrated case;  $p_i$  is the price of the integrated downstream firm in the partially integrated case; and  $p_I$  is the price(s) in the fully integrated case. By the input price effect discussed in Section 3, when  $\tau$  is large enough ( $\tau > \tau_p \simeq 0.709t$ ),  $p_N$  is the lowest among the three cases. In other words, when  $\tau$  is large, vertical integration enhances the downstream prices. From the discussion, I can say that full integration does not increase social welfare and nor will consumers necessarily benefit from integration because full integration leads to maximal product differentiation and enhances the downstream prices. From the figure and Proposition 4, I find that, in most of the range in which partial integration occurs, the non-integrated (*resp.* integrated) downstream firm sets its price higher (*resp.* lower) than one under no integration.

There are two effects generated by a vertical integration: (1) a vertical integration lowers the procurement cost of the integrated downstream firm; (2) as mentioned in Proposition 3, if  $\tau > t/4$ , the procurement cost of the non-integrated downstream firm is increased by integration.

The first effect is related to the following two properties. First, integration directly lowers the downstream price of the integrated firm. Second, because of the strategic complementarity of the downstream pricing, the price of the non-integrated downstream firm decreases. The first effect reduces the downstream prices.

The second effect is related to the following two properties. First, the increment in the procurement cost of the non-integrated downstream firm directly increases the price of it. Second, the increment indirectly increases the price of the integrated downstream firm because of strategic complementarity. To sum up, the non-integrated (*resp.* integrated) downstream firm is directly (*resp.* indirectly) affected by the increase in  $\tau$ . The second effect enhances the downstream prices.

The first and the second effects are trade-offs. As the value of  $\tau$  increases, the second effect, which is related to  $\tau$ , is more significant. The significance of the increment in  $\tau$  on the non-integrated downstream firm is larger than that on the integrated downstream firm.

The implication of the results is briefly discussed. In reality, the degree of product differentiation is not easily evaluated. The results in the paper show that the evaluation of the downstream prices is a useful method to evaluate whether vertical integration is anti-competitive or not. In the model, the increment in the downstream prices is the sufficient condition that vertical integration is anti-competitive. I think that the investigation into the relation between vertical integration(s) and product differentiation provides a new insight in the literature of vertical integration.

## 6 Concluding remarks

This paper presents the development of an equilibrium theory of vertical merger that incorporates strategic behavior in a Hotelling-type location model for the purpose of considering the relation between the strategies of a downstream firm for product differentiation and vertical relation.

I show that vertical integration enhances the degree of product differentiation in the integrated firm. I also show that depending on the upstream firm's transport costs, there



are three patterns of vertical integration: no downstream firm vertically integrates, both downstream firms vertically integrate, and only one downstream firm vertically integrates. In the third case, the profit earned by the non-integrated downstream firm is not always smaller than that in which no vertical integration appears. That is, vertical integration does not always harm the rival downstream firm.

The reason that the non-integrated downstream firm whose costs have been raised does not react with a counter-merger of its own is different from those in Choi and Yi (2000) and Church and Gandal (2000). In both papers, an integrated firm specializes its input for its downstream component. The inputs are unsuitable for its rival downstream firm. The specialization is a commitment device not to supply the rival. In my model, as in the related articles, a commitment to raise the input price facing the rival downstream firm exists. The integrated firm chooses its location at the edge of the Hotelling line. Endogenous product differentiation is an important factor in my paper. The maximum differentiation of the integrated firm mitigates the price competition and then enhances the profits of the non-integrated downstream firm. This is a disincentive to pursue a counter-merger.

In this paper, I assume that upstream firms supply homogenous inputs to downstream firms. If I introduce heterogeneity of their input, the results of the paper may change. This would be a considerable undertaking in a future study. Along this line of enquiry, if each downstream firm has to procure two types of input (for instance, widget and labor), the results could change.<sup>15</sup> This, too, would be a considerable undertaking for a future study.

[2006.3.10, 749 (2006-9)]

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<sup>15</sup> Using a linear demand function, Pepall and Norman (2001) discussed upstream–downstream relations in the case where downstream firms procure multi-inputs from upstream firms.

## APPENDIX

**Proof of Proposition 2:** I check whether or not the location pattern in (17) and (18) is an equilibrium outcome.

First, I consider the case in which  $\tau < t/3$ . Given the location pattern in (17), I check that no firm has an incentive to change its location.

Given the locations in (17), when  $D_1$  locates at  $l_1$ , from (8), the profit of  $D_1$  is:

$$\pi_{d1} = \frac{(1-l_1)(3t-\tau+(t+\tau)l_1)^2}{18t}.$$

The first-order condition is:

$$\frac{\partial \pi_{d1}}{\partial l_1} = \frac{(3t-\tau+(t+\tau)l_1)(3\tau-t-(t+\tau)l_1)}{18t}.$$

If  $\tau < t/3$ , this is negative. Therefore, the optimal location of  $D_1$  is  $l_1 = 0$ .

Given the locations in (17), when  $U_A$  locates at  $h_A$ , from (9), the profit of  $U_A$  is:

$$\pi_{u1} = \frac{\tau(3t-\tau)(1-h_A)(1+h_A)}{6t}.$$

The first-order condition is:

$$\frac{\partial \pi_{u1}}{\partial h_A} = -\frac{h_A(3t-\tau)\tau}{3t}.$$

Therefore, the optimal location of  $U_1$  is  $h_A = 0$ .

Given the locations in (17), when  $D_2$  and  $U_2$  (the integrated downstream and upstream firms) locate at  $l_2$  and  $h_B$ , from (10), the profit of the integrated firm is:

$$\pi_I = \frac{(1-l_2)(3t+\tau+(t+\tau)l_2-2\tau h_B)^2}{18t}.$$

Note that the quantity supplied by  $D_2$  is:

$$(32) \quad 1-x = \frac{3t+\tau+(t+\tau)l_2-2\tau h_B}{6t}.$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial \pi_I}{\partial l_2} &= -\frac{(t-\tau+3(t+\tau)l_2-2\tau h_B)(3t+\tau+(t+\tau)l_2-2\tau h_B)}{18t}, \\ \frac{\partial \pi_I}{\partial h_B} &= -\frac{2\tau(1-l_2)(3t+\tau+(t+\tau)l_2-2\tau h_B)}{9t}. \end{aligned}$$

From the functional form of  $\pi_I$ , I find that  $l_2 = 1$  is not optimal because the profit is zero. Then,  $l_2 < 1$  and the latter partial differential is negative, that is,  $h_B = 0$ . Substituting  $h_B = 0$  into  $\partial\pi_I/\partial l_2$ , I find this to be negative. Therefore, the optimal location of the integrated firm is  $l_2 = h_B = 0$ . Substituting the derived location pattern into (8), (9), (10), (32), and  $w_{A1} = \tau(1 - h_B - l_1)^2$ , I have  $\pi_{d1}$ ,  $\pi_{u1}$ ,  $\pi_I$ ,  $x$ , and  $w_{A1}$  in (17). Using the values, I can easily derive  $p_1$  and  $p_2$  in (17).

Second, I consider the case in which  $\tau \geq t/3$ . Given the location pattern in (18), I check that no firm has an incentive to change its location.

Given the locations in (18), when  $D_1$  locates at  $l_1$ , from (8), the profit of  $D_1$  is:

$$\pi_{d1} = \frac{(1 - l_1)(3t - \tau + (t + \tau)l_1)^2}{18t}.$$

The first-order condition is:

$$\frac{\partial\pi_{d1}}{\partial l_1} = \frac{(3t - \tau + (t + \tau)l_1)(3\tau - t - (t + \tau)l_1)}{18t}.$$

This is zero if  $l_1 = (3\tau - t)/(t + \tau)$ . The optimal location of  $D_1$  is  $l_1 = (3\tau - t)/(t + \tau)$ .

Given the locations in (18), when  $U_A$  locates at  $h_A$ , from (9), the profit of  $U_A$  is:

$$\pi_{u1} = \frac{4\tau(1 - h_A)(5t - 3\tau + 3(t + \tau)h_A)}{27(t + \tau)}.$$

The first-order condition is:

$$\frac{\partial\pi_{u1}}{\partial h_A} = -\frac{8\tau(3\tau - t - (t + \tau)h_A)}{27(t + \tau)}.$$

This is zero if  $h_A = (3\tau - t)/(t + \tau)$ . The optimal location of  $U_1$  is  $h_A = (3\tau - t)/(t + \tau)$ .

Given the locations in (18), when  $D_2$  and  $U_2$  (the integrated downstream and upstream firms) locate at  $l_2$  and  $h_B$ , from (10), the profit of the integrated firm is:

$$\pi_I = \frac{(4t - 3(t + \tau)l_2)(10t + 3(t + \tau)l_2 - 6\tau h_B)^2}{486t(t + \tau)}.$$

Note that the quantity supplied by  $D_2$  is:

$$(33) \quad 1 - x = \frac{10t + 3(t + \tau)l_2 - 6\tau h_B}{18t}.$$

The first-order conditions are:

$$\begin{aligned}\frac{\partial \pi_I}{\partial l_2} &= -\frac{(2t + 9(t + \tau)l_2 - 6\tau h_B)(10t + 3(t + \tau)l_2 - 6\tau h_B)}{162t}, \\ \frac{\partial \pi_I}{\partial h_B} &= -\frac{2\tau(4t - 3(t + \tau)l_2)(10t + 3(t + \tau)l_2 - 6\tau h_B)}{81t(t + \tau)} < 0.\end{aligned}$$

From the functional form of  $\pi_I$ , I find that the latter partial differential is negative. Therefore,  $h_B = 0$ . When  $h_B = 0$ , the former partial differential is also negative. The optimal location of the integrated firm is  $l_2 = h_B = 0$ . Substituting the derived location pattern into (8), (9), (10), (33), and  $w_{A1} = \tau(1 - h_B - l_1)^2$ , I have  $\pi_{d1}$ ,  $\pi_{u1}$ ,  $\pi_I$ ,  $x$ , and  $w_{A1}$  in (18). Using these values, I can easily derive  $p_1$  and  $p_2$  in (18). Q.E.D.

**Proof of Lemma 1:** From (17), (18), and (27), the difference between  $\Pi_I$  and  $\Pi_n (= \pi_{d1} + \pi_{u1})$  is:

$$\Pi_I - \Pi_n = \begin{cases} -\frac{\tau(3t - 2\tau)}{18t} (< 0), & \text{if } \tau \leq t/3, \\ \frac{t(243\tau^2 - 154t\tau - 13t^2)}{486(t + \tau)^2}, & \text{if } \tau > t/3. \end{cases}$$

$\Pi_I - \Pi_n > 0$  if and only if  $\tau > (77 + 8\sqrt{142})t/243$ ;  $\Pi_I - \Pi_n = 0$  if and only if  $\tau = (77 + 8\sqrt{142})t/243$ ;  $\Pi_I - \Pi_n < 0$  if and only if  $\tau < (77 + 8\sqrt{142})t/243$ . From the discussion, I have Lemma 1. Q.E.D.

**Proof of Proposition 3:** From (4), the input prices of the downstream firms,  $\tau(1 - h_B - l_1)^2 = \tau(1 - l_2 - h_A)^2$ , are:

$$(34) \quad w_{A1}^n \equiv \begin{cases} \tau, & \text{if } \tau \leq \frac{t}{4}, \\ \frac{25t^2\tau}{16(t + \tau)^2}, & \text{if } \frac{t}{4} < \tau \leq \frac{3t}{7}, \\ \frac{9t^2(2t + 5\tau - R)^2}{64\tau(t + \tau)^2}, & \text{if } \frac{3t}{7} < \tau, \end{cases}$$

where  $R \equiv \sqrt{4t^2 + 4t\tau + 9\tau^2}$ . Comparing the input price in (34) with that in (17) and (18),

I have

$$w_{A1} - w_{A1}^n = \begin{cases} 0, & \text{if } \tau \leq \frac{t}{4}, \\ \frac{(4\tau - t)(9t + 4\tau)\tau}{16(t + \tau)^2} > 0, & \text{if } \frac{t}{4} < \tau \leq \frac{t}{3}, \\ \frac{31t^2\tau}{144(t + \tau)^2} > 0, & \text{if } \frac{t}{3} < \tau \leq \frac{3t}{7}, \\ \frac{t^2(18t + 77\tau - 9R)(9R - (18t + 13\tau))}{576\tau(t + \tau)^2} > 0, & \text{if } \frac{3t}{7} < \tau. \end{cases}$$

From the discussion, I have Proposition 3.

Q.E.D.

**Proof of Lemma 2:** From (5), (17), and (18), the difference between  $\Pi_i$  and  $\Pi_N (= \pi_{d1} + \pi_{u1})$  is:

$$\Pi_i - \Pi_N = \begin{cases} -\frac{\tau(3t - \tau)}{18t} (< 0), & \text{if } \tau \leq t/4, \\ -\frac{9t^2 - 30t\tau - 2\tau^2}{36t}, & \text{if } t/4 < \tau \leq t/3, \\ \frac{71t^2 - 243t\tau + 486\tau^2}{972(t + \tau)} (> 0), & \text{if } t/3 < \tau \leq 3t/7, \\ \frac{7574t^2 + 15309t\tau + 19683\tau^2}{3888(t + \tau)} - \frac{729(5t + 9\tau)\sqrt{4t^2 + 4t\tau + 9\tau^2}}{3888(t + \tau)} (> 0), & \text{if } 3t/7 < \tau. \end{cases}$$

$\Pi_i - \Pi_N > 0$  if and only if  $\tau > 3(3\sqrt{3} - 5)t/2$ ;  $\Pi_i - \Pi_N = 0$  if and only if  $\tau = 3(3\sqrt{3} - 5)t/2$ ;

$\Pi_i - \Pi_N < 0$  if and only if  $\tau < 3(3\sqrt{3} - 5)t/2$ . From the discussion, I have Lemma 2. Q.E.D.

**Proof of Proposition 5:** The profit of the non-integrated downstream firm in which a partial integration occurs is  $\pi_{d1}$  in (17) if  $\tau \leq t/3$ , otherwise,  $\pi_{d1}$  in (18). The profit of the non-integrated downstream firm in which no integration occurs is  $\pi_{d2}$  in (5). The difference

between them is:

$$\pi_{d1} - \pi_{d2} = \begin{cases} \frac{\tau(6t - \tau)}{18t} (> 0), & \text{if } \tau \leq t/4, \\ \frac{9t^3 - 24t^2\tau + 10t\tau^2 - 2\tau^3}{36t(t + \tau)} (> 0), & \text{if } t/4 < \tau \leq t/3, \\ \frac{t(217t - 486\tau)}{972(t + \tau)} (> 0), & \text{if } t/3 < \tau \leq 3t/7, \\ \frac{t(729\sqrt{4t^2 + 4t\tau + 9\tau^2} - (1024t + 2187\tau))}{1944(t + \tau)}, & \text{if } 3t/7 < \tau. \end{cases}$$

$\pi_{d1} - \pi_{d2} > 0$  if and only if  $\tau < 269297t/588303$ ;  $\pi_{d1} - \pi_{d2} = 0$  if and only if  $\tau = 269297t/588303$ ;  $\pi_{d1} - \pi_{d2} < 0$  if and only if  $\tau > 269297t/588303$ . From the discussion, I have Proposition 5. Q.E.D.

**Social surplus and consumer surplus:** To derive Propositions 6 and 7, I calculate social surplus and consumer surplus in three cases: no integration, partial integration, and full integration. I use the equations in (28) and (29).

First, I calculate social surplus and consumer surplus in the no integration case. When  $\tau \leq t/4$ , social welfare and consumer surplus are:

$$(35) \quad SW = s - \int_0^{\frac{1}{2}} t(x - l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1 - l_2 - x)^2 dx - \frac{1}{2}\tau(l_1 - h_A)^2 - \frac{1}{2}\tau(l_2 - h_B)^2 \\ = s - \frac{t}{12},$$

$$(36) \quad CS = s - \int_0^{\frac{1}{2}} t(x - l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1 - l_2 - x)^2 dx - \frac{p_1}{2} - \frac{p_2}{2} \\ = s - \frac{13t + 12\tau}{12}.$$

When  $t/4 < \tau \leq 3t/7$ , social welfare is:

$$(37) \quad SW = s - \int_0^{\frac{1}{2}} t(x - l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1 - l_2 - x)^2 dx - \frac{1}{2}\tau(l_1 - h_A)^2 - \frac{1}{2}\tau(l_2 - h_B)^2 \\ = s - \frac{13t^2 - 44t\tau + 48\tau^2}{48(t + \tau)},$$

$$(38) \quad CS = s - \int_0^{\frac{1}{2}} t(x - l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1 - l_2 - x)^2 dx - \frac{p_1}{2} - \frac{p_2}{2} \\ = s - \frac{5t(17t - 4\tau)}{48(t + \tau)}.$$

When  $3t/7 < \tau$ , social welfare is:

$$(39) \quad SW = s - \int_0^{\frac{1}{2}} t(x - l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1 - l_2 - x)^2 dx - \frac{1}{2}\tau(l_1 - h_A)^2 - \frac{1}{2}\tau(l_2 - h_B)^2$$

$$= s - \frac{(108t^3 + 494t^2\tau + 845t\tau^2 + 972\tau^3) - 9(6t^2 + 23t\tau + 36\tau^2)\sqrt{4t^2 + 4t\tau + 9\tau^2}}{96\tau(t + \tau)},$$

$$(40) \quad CS = s - \int_0^{\frac{1}{2}} t(x - l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1 - l_2 - x)^2 dx - \frac{p_1}{2} - \frac{p_2}{2}$$

$$= s - \frac{t(108t^2 + 278t\tau + 89\tau^2 - 27(2t + \tau)\sqrt{4t^2 + 4t\tau + 9\tau^2})}{96\tau(t + \tau)}.$$

Second, I calculate social surplus and consumer surplus under the partial integration case. From (17), if  $\tau < t/3$ , consumer surplus and social welfare are:

$$CS = s - \int_0^{\frac{3t-\tau}{6t}} t(x - l_1)^2 dx - \int_{\frac{3t-\tau}{6t}}^1 t(1 - l_2 - x)^2 dx - \frac{3t - \tau}{6t}p_1 - \frac{3t + \tau}{6t}p_2$$

$$= s - \frac{39t^2 + 18t\tau - \tau^2}{36t}.$$

$$SW = s - t \int_0^{\frac{3t-\tau}{6t}} x^2 dx - t \int_{\frac{3t-\tau}{6t}}^1 (1 - x)^2 dx = s - \frac{3t^2 + \tau^2}{36t}.$$

From (18), if  $\tau \geq t/3$ , consumer surplus and social welfare are:

$$CS = s - \int_0^{\frac{4}{9}} t(x - l_1)^2 dx - \int_{\frac{4}{9}}^1 t(1 - l_2 - x)^2 dx - \frac{4p_1}{9} - \frac{5p_2}{9}$$

$$= s - \frac{t(377t + 81\tau)}{243(t + \tau)}.$$

$$SW = s - t \int_0^{\frac{4}{9}} \left(x - \frac{3\tau - t}{3(t + \tau)}\right)^2 dx - t \int_{\frac{4}{9}}^1 (1 - x)^2 dx$$

$$= s - \frac{t(49t^2 - 62t\tau + 81\tau^2)}{243(t + \tau)^2}.$$

I summarize the calculations as follows:

$$(41) \quad CS = \begin{cases} s - \frac{39t^2 + 18t\tau - \tau^2}{36t}, & \text{if } \tau < t/3, \\ s - \frac{t(377t + 81\tau)}{243(t + \tau)}, & \text{if } \tau \geq t/3, \end{cases}$$

$$(42) \quad SW = \begin{cases} s - \frac{3t^2 + \tau^2}{36t}, & \text{if } \tau < t/3, \\ s - \frac{t(49t^2 - 62t\tau + 81\tau^2)}{243(t + \tau)^2}, & \text{if } \tau \geq t/3. \end{cases}$$

Finally, I calculate social surplus and consumer surplus in the full integration case. From (27), I derive consumer surplus and social welfare:

$$(43) \quad CS = s - \int_0^{\frac{1}{2}} t(x - l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1 - l_2 - x)^2 dx - \frac{p_1}{2} - \frac{p_2}{2} = s - \frac{13t}{12},$$

$$(44) \quad SW = s - t \int_0^{\frac{1}{2}} x^2 dx - t \int_{\frac{1}{2}}^1 (1 - x)^2 dx = s - \frac{t}{12}.$$

**Proof of Proposition 6:** From (35), (37), (39), and (44),  $SW_N - SW_F$  is:

$$\begin{cases} 0, & \text{if } \tau \leq t/4, \\ \frac{(4\tau - t)(3t - 4\tau)}{16(t + \tau)} (> 0), & \text{if } t/4 < \tau \leq 3t/7, \\ \frac{3(6t^2 + 23t\tau + 36\tau^2)\sqrt{4t^2 + 4t\tau + 9\tau^2}}{32\tau(t + \tau)} - \frac{3(12t^3 + 54t^2\tau + 93t\tau^2 + 108\tau^3)}{32\tau(t + \tau)} (> 0), & \text{if } 3t/7 < \tau. \end{cases}$$

$SW_N = SW_F$  if and only if  $\tau \leq t/4$ .  $SW_N > SW_F$  if and only if  $\tau > t/4$ .

From (35), (37), (39), and (42),  $SW_N - SW_P$  is:

$$\begin{cases} \frac{\tau^2}{36t} (> 0), & \text{if } \tau \leq t/4, \\ \frac{-27t^3 + 144t^2\tau - 140t\tau^2 + 4\tau^3}{144t(t + \tau)} (> 0), & \text{if } t/4 < \tau \leq t/3, \\ \frac{-269t^3 + 1519t^2\tau + 972t\tau^2 - 3888\tau^3}{3888(t + \tau)^2} (> 0), & \text{if } t/3 < \tau \leq 3t/7, \\ \frac{729(t + \tau)(6t^2 + 23t\tau + 36\tau^2)\sqrt{4t^2 + 4t\tau + 9\tau^2}}{7776\tau(t + \tau)^2} - \frac{8748t^4 + 47194t^3\tau + 110443t^2\tau^2 + 144585t\tau^3 + 78732\tau^4}{7776\tau(t + \tau)^2} (> 0), & \text{if } 3t/7 < \tau. \end{cases}$$

$SW_N > SW_P$  for any  $\tau \in [0, t]$ .

From (42) and (44),  $SW_F - SW_P$  is:

$$\begin{cases} \frac{\tau^2}{36t} (> 0), & \text{if } \tau \leq t/3, \\ \frac{t(115t^2 - 410t\tau + 243\tau^2)}{972(t + \tau)^2}, & \text{if } t/3 < \tau. \end{cases}$$

$SW_F > SW_P$  if and only if  $\tau < (205 - 16\sqrt{55})t/243$ ;  $SW_F = SW_P$  if and only if  $\tau = (205 - 16\sqrt{55})t/243$ ;  $SW_F < SW_P$  if and only if  $\tau > (205 - 16\sqrt{55})t/243$ . From the discussion, I have Proposition 6. Q.E.D.



**Proof of Proposition 7:** From (36), (38), (40), and (43),  $CS_N - CS_F$  is:

$$\left\{ \begin{array}{ll} -\tau (< 0), & \text{if } \tau \leq t/4, \\ \frac{(24\tau - 11t)t}{16(t + \tau)} (< 0), & \text{if } t/4 < \tau \leq 3t/7, \\ \frac{9(2t + \tau)\sqrt{4t^2 + 4t\tau + 9\tau^2}}{32\tau(t + \tau)} - \frac{36t^2 + 58t\tau - 5\tau^2}{32\tau(t + \tau)}, & \text{if } 3t/7 < \tau. \end{array} \right.$$

$CS_N > CS_F$  if and only if  $\tau > (25\sqrt{1417} - 779)t/352$ ;  $CS_N = CS_F$  if and only if  $\tau = (25\sqrt{1417} - 779)t/352$ ;  $CS_N < CS_F$  if and only if  $\tau < (25\sqrt{1417} - 779)t/352$ .

From (36), (38), (40), and (41),  $CS_N - CS_P$  is:

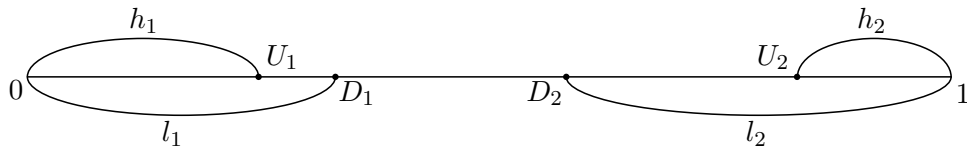
$$\left\{ \begin{array}{ll} -\frac{\tau(18t + \tau)}{36t} (< 0), & \text{if } \tau \leq t/4, \\ \frac{-99t^3 + 288t^2\tau + 68t\tau^2 - 4\tau^3}{144t(t + \tau)}, & \text{if } t/4 < \tau \leq t/3, \\ \frac{(2916\tau - 853t)t}{3888(t + \tau)} (> 0), & \text{if } t/3 < \tau \leq 3t/7, \\ \frac{2187(2t + \tau)\sqrt{4t^2 + 4t\tau + 9\tau^2}}{7776\tau(t + \tau)} - \frac{t(8748t^2 + 10454t\tau + 4617\tau^2)}{7776\tau(t + \tau)} (> 0), & \text{if } 3t/7 < \tau. \end{array} \right.$$

$CS_N > CS_P$  if and only if  $\tau > \bar{\tau}$ ;  $CS_N = CS_P$  if and only if  $\tau = \bar{\tau}$ ;  $CS_N < CS_P$  if and only if  $\tau < \bar{\tau}$ .

From (41) and (43),  $CS_F - CS_P$  is:

$$\left\{ \begin{array}{ll} \frac{\tau(18t - \tau)}{36t} (> 0), & \text{if } \tau \leq t/3, \\ \frac{t(455t - 729\tau)}{972(t + \tau)}, & \text{if } t/3 < \tau. \end{array} \right.$$

$CS_F > CS_P$  if and only if  $\tau < 455t/729$ ;  $CS_F = CS_P$  if and only if  $\tau = 455t/729$ ;  $CS_F < CS_P$  if and only if  $\tau > 455t/729$ . From the discussion, I have Proposition 7. Q.E.D.



**Figure 1: The locations of the firms**

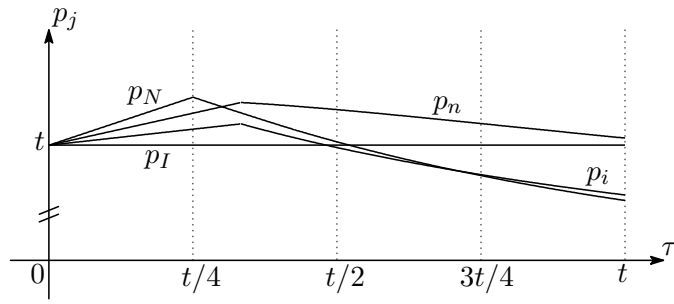


Figure 2: The downstream prices in the three cases

		Firm 1	
		No integration	Integration
Firm 2	No integration	$\Pi_N, \Pi_N$	$\Pi_n, \Pi_i$
	Integration	$\Pi_i, \Pi_n$	$\Pi_I, \Pi_I$

**Table 1: Payoff matrix for the first stage (vertical integration) game**

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